Transport Phenomena

FORCED CONVECTION

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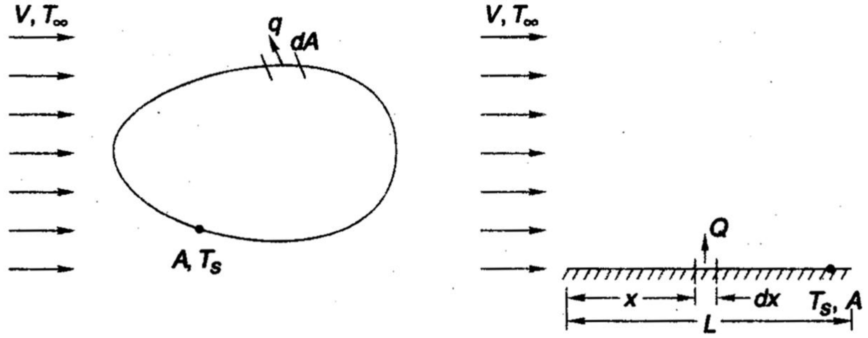
Forced Convection

Convection is the mode of heat transfer between a surface and a fluid moving over it. The energy transfer in convection is predominantly due to the bulk motion of the fluid particles, though the molecular conduction within the fluid itself also contributes to some extent. If this motion is mainly due to the density variations associated with temperature gradients or natural convection. On the other hand, if this fluid motion is principally produced by some superimposed velocity field (like a fan, blower or a pump), the energy transport is said to be due to forced convection.

THE CONVECTIVE HEAT TRANSFER COEFFICIENT

Consider following figure.



Local heat flux, ‘q’ for an arbitrarily shaped surface of area A at temperature T_S, over which flows a fluid of velocity V and of temperature T_∞ is given by

$$q = h(T_S - T_\infty) \qquad (1)$$

Where ‘h’ is the local heat transfer coefficient. This equation is referred to as Newton’s law of cooling. Due to the variation of flow conditions from point to point, the value of q and h along the surface also vary and that is why the adjective local is applied to them. The total heat transfer rate may be obtained by integrating eq 1 over the entire surface, assuming a uniform value of T_S.

$$Q = \int q.dA = (T_S - T_\infty)\int h.dA \qquad (2)$$

Defining $\bar{h}$ as the average or total heat transfer coefficient for the entire surface, we may write equation 2 as,

$$Q = \bar{h}A(T_S - T_\infty) \qquad (3)$$

Comparing eq. 2 and eq. 3, the average and local convection coefficients are related by

$$\bar{h}_A = (1/A)\int h.dA \qquad (4)$$

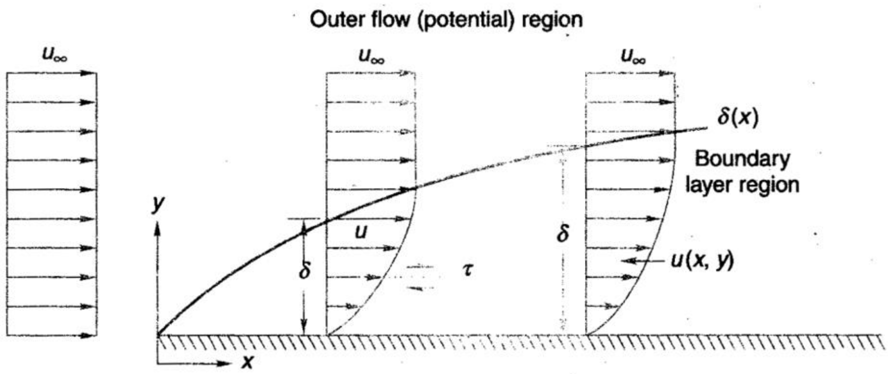
Note that in the case of a simple flat plate of unit width where `h' is a function of x alone,

$$\bar{h}_L = \left(\frac{1}{L}\right)\int_0^h h \; . \; dx \qquad (5)$$

Invariably, the problem of convection is to determine the local heat flux or the total heat transfer rate, which may be determined from eq.1 and 3 provided that the local and total heat transfer coefficients are known. The aim of any convection analysis is to get these coefficients first.

THE BOUNDARY LAYER CONCEPT

Consider the flow over a flat plate of following figure.



The velocity here in front of the **leading edge** of the plate is **uniform**. Due to the no slip condition to be satisfied at the surface of the plate, the velocity of the fluid is reduced to zero relative to the surface. This results in the retardation of the fluid particles in the adjoining fluid layers until at a distance y = δ from the surface (called **boundary layer thickness**) this effect becomes negligible.

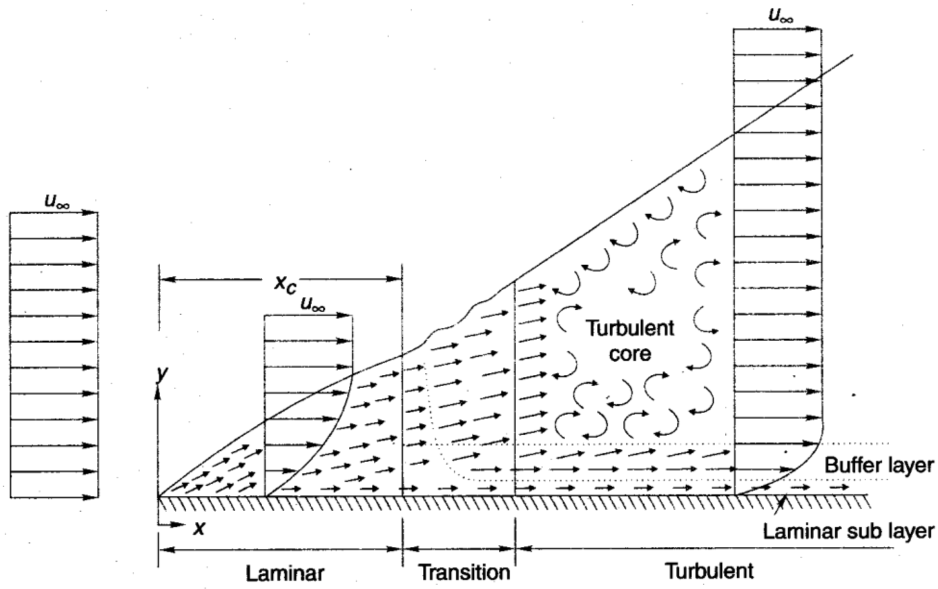
The **deceleration** of the fluid particle in the boundary layer is associated with **shear stress**, τ. The effect of the shear (viscous) forces originating at the surface extends into the body of the fluid, but with increasing distance, y, from the surface, the x velocity component of the fluid, u, increases until it approaches the free stream velocity, $\mathbf{u}_\infty$. The effects of viscosity penetrate further into the free stream resulting in the growth of a boundary layer downstream. The boundary layer thickness, **δ** is defined as the value of y for which u = 0.99 .

Because of the large velocity gradient across the flow the frictional shearing stresses ($\tau = \mu \, \partial u/\partial y$) in the boundary layer are quite large even for fluids with low viscosity. On the other hand, these stresses are very small outside the boundary layer. Thus the flow field can be conveniently divided into two regions:

- *a thin region near the body*, called the boundary layer, where the velocity and temperature gradients are large and
- *the region outside the boundary layer* where velocity and temperature gradients are very nearly equals to their free stream values. The thickness of boundary layer has been arbitrarily defined as the distance from the surface at which the local velocity (or

temperature) reaches 99 % of the external velocity (or temperature). In general, both the velocity boundary layer and thermal boundary layer will exist simultaneously.

It is most essential to **distinguish** between laminar and turbulent boundary layers. Initially, the boundary layer development is laminar as shown in following figure for the flow over a flat plate.



Depending upon the flow field and fluid properties, at some **critical distance** from the leading edge small disturbances in the flow begin to get amplified, a **transition** process takes place and the flow becomes **turbulent**.

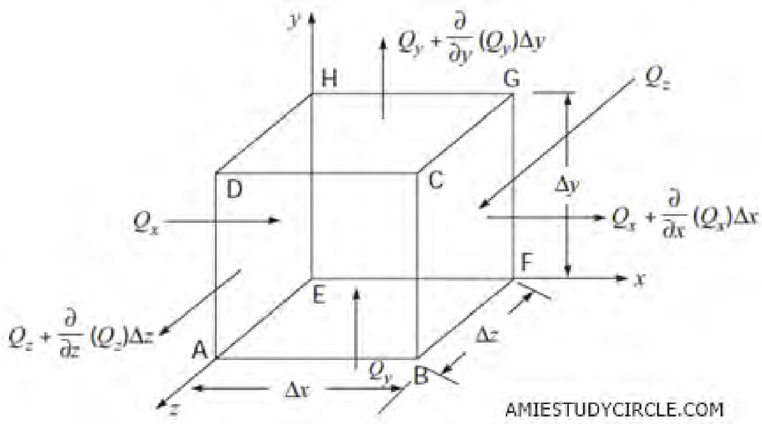
In laminar boundary layer, the fluid motion is highly ordered whereas the motion in the turbulent boundary layer is highly irregular with the fluid moving to and from in all directions. Due to fluid mixing resulting from these macroscopic motions, the turbulent boundary layer is thicker and the velocity profile in turbulent boundary layer is flatter than that in laminar flow.

The critical distance, x_c, beyond which the flow cannot retain its laminar character is usually specified in terms of a critical Reynolds number $Re_{x,c}$. Depending upon surface and turbulence level of free stream the critical Reynolds number varies between 10^5 and 3 x 10^6.

A value of $Re_{x,c} = \frac{\rho . u_\infty . x_c}{\mu} = 5x10^5$ (6)

ENERGY EQUATION ON FLAT PLATE

Let us consider a small fluid element in motion as a closed system as shown in figure.



Consider following equation

$$\frac{dQ}{dt} = \frac{dE}{dt} + \frac{dW}{dt} \quad (1)$$

Consider a differential element. We get following heat equation

$$\frac{dQ}{dt} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)dxdy \quad (2)$$

Also

$$\frac{dE}{dt} = \rho\left[\frac{De}{Dt} + \frac{1}{2}\frac{D}{Dt}(u^2 + v^2)\right]dxdy \quad (3)$$

Total work done by body force

$$\frac{dW_B}{dt} = (B_x u + B_y v)dxdy \quad (4)$$

Now, the rate of work performed on the element due to the normal stress σ_{xx}, is given by

$$\frac{\partial}{\partial x}(u\sigma_{xx})dxdy$$

and that due to normal stress σ_{yy} is

$$\frac{\partial}{\partial y}(v\sigma_{yy})dxdy$$

Similarly, the rate of work done due to shear stresses, τ_{xx} and τ_{yy} are

$$\frac{\partial}{\partial x}(v\tau_{xy})dxdy$$

and

$$\frac{\partial}{\partial y}(u\tau_{yx})dxdy$$

Total work done due to friction

$$\frac{dW_f}{dt} = \left[\frac{\partial}{\partial x}(u\sigma_{xx} + v\tau_{xy}) + \frac{\partial}{\partial y}(v\sigma_{yy} + u\tau_{yx}\right]dxdy \quad (5)$$

Therefore total work done by particle

$$\frac{dW}{dt} = \frac{-dW_f}{dt} - \frac{dW_B}{dt}$$

$$= -\left[uB_x + vB_y + \frac{\partial}{\partial x}(u\sigma_{xx} + v\tau_{xy}) + \frac{\partial}{\partial y}(v\sigma_{yy} + u\tau_{yx})\right]dxdy \qquad (6)$$

Putting (2), (3) and (6) into eqn (1), we finally get

$$\rho\frac{De}{Dt} = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\phi \qquad (7)$$

where ϕ is viscous energy dissipation.

For incompressible fluid

$$\frac{De}{Dt} = c_p\frac{Dt}{Dt}$$

From this energy equation becomes

$$\rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\phi \qquad (8)$$

In engineering fluid flow problems, $\mu\phi = 0$

$$\therefore \quad \rho c_p\left(u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (9)$$

Navier Stokes equations along with continuity equation reduce to

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (10)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{dp}{dx} + v\frac{\partial^2 u}{\partial y^2} \qquad (11)$$

and

$$\frac{\partial p}{\partial y} = 0 \qquad (12)$$

Equations (10) and (11) are known as **Prandtl's boundary layer equations**. These are to be solved subject to the following boundary conditions:

y = 0; u = 0; v = 0; y = ∞; u=U(x) (13)

The energy equation (9) reduces to

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \frac{v}{C_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (14)$$

The boundary conditions are

$y = 0\ T = T_s$

$y = \infty,\ T = T_\infty$

DIMENSIONAL ANALYSIS

Dimensional Analysis Applied to Forced Convection

Let us consider the case of a fluid flowing across a heated tube. The various variables pertinent to this problem along with their symbols and dimensions are given in Table 1

Table 1: Pertinent variables in forced convection heat transfer

S. No.	Variable	symbol	Dimension
1.	Tube diameter (Characteristic length)	D	L
2.	Fluid density	ρ	$M\ L^{-3}$
3.	Fluid velocity	U	$L\ t^{-1}$
4.	Fluid viscosity	μ	$M\ L^{-1} t^{-1}$
5.	Specific heat	C_p	$L^2 t^{-2} T^{-1}$
6.	Thermal conductivity	k	$M\ L\ t^{-3} T^{-1}$
7.	Heat transfer coeff.	h	$M\ t^{-3} T^{-1}$

There are seven variables and four basic dimensions, so three independent dimensionless parameters would be required to correlate the experimental data.

The three dimensionless groups will be symbolized by π_1, π_2 and π_3 and may be obtained by set of procedure. Each dimensionless parameter would be formed by combining core group of 'r' variables with one of the remaining variables not in core. The will include any four (in this case) of the variables which among them include all of the basic dimensions. We may, arbitrarily choose D, ρ , π and k as the core. The groups to be formed are now represented as the following π groups

$$\pi_1 = D^a \rho^b \mu^c\ k^d U$$
$$\pi_1 = D^e \rho^f \mu^g\ k^h C_p$$
$$\pi_1 = D^j \rho^l \mu^m\ k^n h$$

Since these groups are to be dimensionless, so the variables are raised to certain exponents, a, b, c, m, n. Starting with π_1, we write dimensionally as

$$M^0\ L^0\ T^0\ t^0\ =\ 1\ =\ (L)^a \left(\frac{M}{L^3}\right)^b \left(\frac{M}{Lt}\right)^c \left(\frac{ML}{t^3 T}\right)^d \left(\frac{L}{t}\right)$$

Equating the sum of the exponents of each basic dimension to zero, we get the following set of equations

for;

M; $\quad 0 = b + c + d$

L; $0 = a - 3b + d + 1 + c$

t; $0 = - c - 3d - 1$

T; $0 = - d$

Solving these equations, we get

d = 0

c = -1

b = 1

a = 1

giving $$\pi_1 = \left(\frac{\rho UD}{\mu}\right) = Re_D$$ (Reynolds's Number)

Similarly for π_2

$$1 = (L)^e \left(\frac{M}{L^3}\right)^f \left(\frac{M}{Lt}\right)^g \left(\frac{ML}{t^3 T}\right)^i \left(\frac{L^2}{t^2 T}\right)$$

for

M; $0 = f + g + i$

L; $0 = e - 3f - g + i + 2$

t; $0 = - g - 3i - 2$

T; $0 = - i - 1$

from these we may find that i = 1, g = 1, f = 0, e = 0, giving

$$\pi_2 = \frac{\mu C_p}{k} = Pr$$ (Prandtl number)

By following a similar procedure, we can obtain

$$\pi_3 = \frac{hd}{k} = Nu$$ (Nusselt number)

Now F(π_1, π_2, π_3) as Nu = ϕ (Re, Pr) (1)

It is worthwhile to point out here that we chose the core variables quite arbitrary. Had we chosen a different core group in our dimensional analysis, viz. D, ρ, μ, C_p, the π group obtained would have been Re, Pr and a non-dimensional form of heat transfer coefficient which is designated as Stanton number St, and is expressed as

$$St = \frac{Nu}{Re.Pr} = \frac{h}{\rho.U.C_p}$$

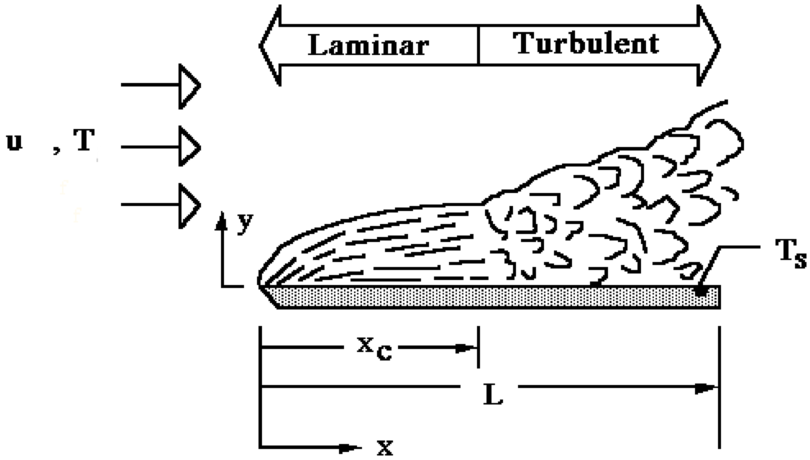
So another form of correlating heat transfer data is

St = ϕ (Re, Pr) (2)

PARALLEL FLOW OVER A FLAT PLATE

Laminar Boundary Layer calculations

Parallel flow over a flat plate is the simplest case of boundary layer development in external flow.



Defining the boundary layer thickness, δ, as the value of y for which $u/u_\infty = 0.99$,

$$\delta = \frac{5x}{\sqrt{Re_x}} \tag{17}$$

Local skin friction coefficient is

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} \tag{18}$$

Local Nusselt number, Nu_x is given by

$$Nu_x = \frac{h_x x}{k} = 0.332\, Re_x^{1/2}\, Pr^{1/3} \tag{19}$$

where h_x is local heat transfer coefficient.

$$h_x = 0.331\left(\frac{k}{x}\right) Re_x^{1/2}\, Pr^{1/3}$$

Average heat transfer coefficient is

$$\bar{h}_x = 2h_x$$

The average value of Nusselt number is therefore

$$\overline{Nu}_x = \frac{\bar{h}_x x}{k} = 0.664\, Re_x^{1/2}\, Pr^{1/3} \qquad (Pr \geq 0.6) \tag{20}$$

The fluid properties involved in these expressions are normally evaluated at the film temperature, T_f, defined as

$$T_f = \frac{T_s + T_\infty}{2} \tag{21}$$

Example

Air at 20^{0}C is flowing along a heated flat plate at 134^{0}C at a velocity of 3 m/s. The plate is 2 m long and 1.5 m wide. Calculate the thickness of the hydrodynamic boundary layer and skin *friction coefficient at 40 cm from the leading edge of the plate. The kinematic viscosity of air at 20^{0}C may be taken as 15.06 x 10^{-6} m^2/s.*

Solution

At x = 40 cm; Re = $u_\infty x/\nu$ = (3)(0.4)/(15.06 x 10^{-6}) = 7.9 x 10^4 < 5 x 10^5

so the boundary layer is laminar. Its thickness is calculated from Eq. 17

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{(5)(0.4)}{(7.9x10^4)^{1/2}} = 0.71x10^{-2}\,m = 7.1\,mm$$

The local skin friction coefficient is given by equation 18

$$C_{fx} = \frac{0.664}{\sqrt{Re_x}} = \frac{0.664}{(7.9x10^4)^{1/2}} = 2.36x10^{-3}$$

Example

For the flow system in previous example, calculate the local heat transfer coefficient at x = 0.4 m and the heat transferred from the first 40 cm of the plate.

Solution

Given data

The film temperature, $T_f = \frac{134+20}{2} = 77^0\,C$

The physical properties of air at 77^{0}C are

ρ = 0.998 kg/m^3, C_p = 1.009 kJ/kg^{0}C, ν = 20.76 x 10^{-6} m^2/s, k = 0.03 W/mK, Pr = 0.697

x = 0.4 m

Reynolds and Nusselt numbers

$$Re_x = \frac{u_\infty x}{\nu} = \frac{(3)(0.4)}{20.76x10^{-6}} = 5.78x10^4$$

$$Nu_x = \frac{h_x x}{k} = 0.332\,Re_x^{1/2}\,Pr^{1/3} = (0.332)(5.78x10^4)(0.697)^{1/3} = 70.6$$

Heat transfer coefficient

$$h_x = (2)(5.3) = 10.6\,W/m^2K$$

Heat flow

The heat flow is

$$Q = \overline{h}_x A(T_s - T_\infty) = (10.6)(0.4)(1.5)(134-20) = 725\,W$$

The heat flow from the both sides of the plate = (2)(725) = 1450 W

Problem

Air at 27^{0}C flows over a flat at a velocity of 2 m/s. The plate is heated over its entire length to a temperature of 60^{0}C. Calculate the heat transfer for the first 20 cm of the plate. The properties of air at 43.5^{0}C are: ν = 17.36 x 10^{-6} m^2/s, k = 0.0275 W/mK, Pr = 0.7, C_p = 1.006 kJ/kg.^{0}C.

Answer: 8.18 W/m

ANALOGY BETWEEN MOMENTUM AND ENERGY TRANSFER: COLBURN AND REYNOLDS ANALOGY

The ratio of heat flux to shear stress in laminar flow is given by

$$\frac{q_s}{\tau_s} = -\frac{k\frac{dT}{dy}}{\mu\frac{du}{dy}} = -\frac{k}{\mu}\frac{dT}{du} = -\frac{C_p}{P_r}\frac{dT}{du}$$

where Pr = C_P/k

For the identical velocity and thermal boundary layers

Pr = 1

$\therefore$ $$\frac{q_s}{\tau_s} = -C_p\frac{dT}{du}$$

or $$-\frac{q_s}{C_p\tau_s}\int_0^{u_\infty} du = \int_{T_s}^{T_\infty} dT$$

$$\frac{q_s}{C_p\tau_s}\mu_\infty = T_s - T_\infty$$

$$\frac{q_s}{(T_s - T_\infty)}\cdot\frac{1}{\rho C_p u_\infty} = \frac{\tau_s}{\rho u_\infty^2} \quad (1)$$

Now with $C_{fx} = \frac{2\tau_s}{\rho u_\infty^2}$; C_{fx} is skin friction.

and $$h_x = \frac{q_s}{T_s - T_\infty} \quad (2)$$

The dimensionless group of terms on the left side of eq. 2 is called **Stanton number**, St and the **Nusselt number** divided by the product of the Reynolds and Prandtl number, i.e.

$$\frac{N_u}{\mathrm{Re}_x \mathrm{Pr}} = St_x = \frac{C_{fx}}{2} \quad (3)$$

This equation (3) is called **Reynolds analogy**. This expresses the relationship between fluid friction and heat transfer for **laminar flow on a flat plate**.

☞ ***Reynolds analogy*** *is valid only when the Prandtl number is unity. If the Prandtl number is different than 1, then we have to use* ***Colburn analogy****.*

Skin friction is given by

$$C_{fx} = \frac{0.664}{\sqrt{\mathrm{Re}_x}}$$

or $$\frac{C_{fx}}{2} = 0.332\,\mathrm{Re}_x^{-(1/2)} \qquad (4)$$

Also $$Nu_x = 0.332\,\mathrm{Re}_x^{1/2}\,\mathrm{Pr}^{1/3}$$

Dividing the above equation by ($\mathrm{Re}_x \mathrm{Pr}^{1/3}$), we get

$$\frac{Nu_x}{\mathrm{Re}_x\,\mathrm{Pr}^{1/3}} = \left(\frac{Nu_x}{\mathrm{Re}_x\,\mathrm{Pr}}\right)\mathrm{Pr}^{2/3} = 0.332\,\mathrm{Re}_x^{(-1/2)}$$

$$St_x\,\mathrm{Pr}^{2/3} = 0.332\,\mathrm{Re}_x^{-1/2} \qquad (5)$$

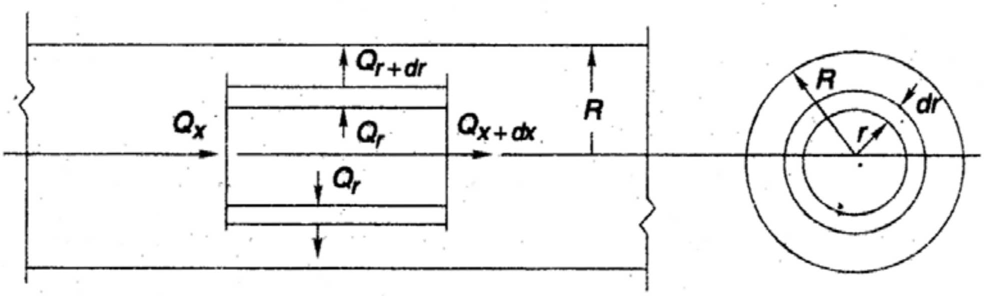
From (4) and (5)

$$St_x\,\mathrm{Pr}^{2/3} = \frac{C_{fx}}{2} \qquad (6)$$

☞ *This equation is known as* ***Colburn analogy****. This analogy is valid for many heat transfer situations for 0.5 < Pr < 50.*

FULLY DEVELOPED LAMINAR FLOW IN CIRCULAR TUBES

The temperature distribution and convection heat transfer for a fully developed laminar flow can be determined from the solution of energy equation subject to appropriate boundary conditions.



The energy equation for a fully developed flow in a tube is reduced to

$$\frac{1}{\alpha}u\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}$$

where α = k/ρC_p is the thermal diffusivity of the fluid.

The radial heat flow conducted into the annular element is

$$dQ_r = -k(2\pi r dx)\frac{\partial T}{\partial r}$$

and the radial heat conducted out is

$$dQ_{r+dr} = -k2\pi(r+dr)dx\left(\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial r^2}dr\right)$$

Similarly the axial heat flow in and out are

$$dQ_x = -k(2\pi r dr)\frac{\partial T}{\partial x}$$

$$dQ_{x+dx} = -k(2\pi r dr)\left(\frac{\partial T}{\partial x} + \frac{\partial^2 T}{\partial x^2}dx\right)$$

The net heat convected out of the element is

$$2\pi r.dr\rho C_p u.\frac{\partial T}{\partial x}dx$$

The energy balance equation is:

Net energy conducted in = net energy convected out

We have $$\frac{1}{\alpha}u\frac{\partial T}{\partial x} = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial x^2}$$

This equation can be written as

$$\frac{1}{\alpha}u\frac{\partial T}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial x^2}$$

Constant Wall Heat Flux

$$\mathrm{Nu_D} = \frac{\mathrm{hD}}{\mathrm{k}} = 4.364$$

Constant Wall Temperature

$$\mathrm{Nu_D} = 3.66$$

Example

A simple water heating system consists of a thick walled tube of I.D. 25 mm and O.D. 40 mm. Water at the rate of 5 kg/min enters at 20^0C and leaves at 70^0C. Electrical heating within the wall of the tube generates heat at a uniform rate of 10^7 W/m^3. Calculate the length of the tube. Also compute the heat transfer coefficient at the outlet if the inner surface temperature of the tube is 80^0C.

Solution

Calculations for length

Assuming that the whole of the heat generated in the tube wall is utilized for heating water, the energy balance for a length L, of tube is

$$q\frac{\pi}{4}(D_0^2 - D_i^2)L = mC_p(T_{b,0} - T_{b,i})$$

C_p of water at T_b = (20 + 70)/2 = 45^{0}C = 4.179 kJ/kg.K

or $$L = \frac{4mC_p}{(D_0^2 - D_i^2)q}(T_{b,0} - T_{b,i}) = \frac{(4)(5/60)(4179)}{\pi(0.04^2 - 0.025^2)(10^7)}(70 - 25) = 2.43\,m$$

A length of 2.43 m is required to achieve the desired outlet temperature of water.

Local convection coefficient

The local convection coefficient at the outlet of the tube is

$$h_0 = \frac{q_s}{(T_{s,0} - T_{b,0})}$$

where q_s, the heat flux at the surface is obtained from

$$q.\frac{\pi}{4}(D_0^2 - D_i^2)L = q_s \pi D_i L$$

or $$q_s = \frac{q}{4}.\frac{(D_0^2 - D_i^2)}{D_i} = \frac{10^7(0.04^2 - 0.02^2)}{(4)(0.025)} = 1x10^5\ W/m^2$$

$$h_0 = \frac{1x10^5}{(80 - 70)} = 1x10^4\ W/m^2K$$

Example

Estimate the heat transfer coefficient for a laminar fully developed fluid (k = 0.175 W/mK) inside a 6 mm I.D. tube under uniform wall temperature boundary conditions. Also compute the heat transfer between the tube walls and the fluid for a length of 8 m if the mean temperature difference between the wall and the fluid is 50^{0}C.

Solution

$$Nu_D = \frac{hD}{k} = 3.66$$

or $$h = 3.66\frac{k}{D} = \frac{(3.66)(0.175)}{(0.006)} = 106.75\,W/m^2K$$

Now $$Q = h(\pi DL)\Delta T = (106.75)(\pi x0.006x8)(50) = 805\,W$$

Example

Air at one atmospheric pressure and 75°C enters a tube of 4.0 mm internal diameter with an average velocity of 2 m/s. The tube length is 1.0 m and a constant heat flux is Imposed by the tube surface on the air over the entire length. An exit bulk mean temperature of air of 125°C is required. Determine (a) the heat transfer coefficient at exit h_L, *(b) the constant surface heat flux* q_w, *and (c) the exit tube surface temperature. The properties of air at the average temperature of inlet and outlet bulk mean temperatures, i.e. (75 + 125)/2 = 100°C are as follows:*

$\rho = 0.95\ kg/m^3$

$\mu = 2.18 \times 10^{-5} kg/(ms)$

$c_p = 1.01\ kJ/(kg\ K)$

$k = 0.03 W/(mK)$

$Pr = 0.70$

Solution

Type of flow

$$Re = \frac{0.95x2x4x10^{-3}}{2.18x10^{-5}} = 349$$

Therefore the flow is laminar (< 2500).

Heat transfer coefficient

For a fully developed flow with constant surface heat flux, Nu = 4.36.

$$h = Nu\left(\frac{k}{D}\right) = 4.36\left(\frac{0.03}{0.004}\right) = 32.7W/m^2K$$

Exit temperature

From an overall rate of air is given by

$$q_w \pi DL = mc_p(T_{b,o} - T_{b,i})$$

The mass flow rate of air is given by

$$m = (0.95)\left(\frac{\pi}{4}\right)(4x10^{-3})^2 x2 = 2.39x10^{-5} kg/s$$

$$\therefore \quad q_w = \frac{(2.39x10^{-5})(1.01x10^3)(125-75)}{\pi(4x10^{-3})(1.0)} = 96W/m^2$$

Let $T_{w,e}$ be the tube temperature at the exit plane. Then, we can write

$$h_L(T_{w,e} - T_{b,o}) = q_w$$

or $$T_{w,e} = T_{b,o} + \frac{q_w}{h_L} = 125 + \frac{96}{32.7} = 128^0 C$$

FULLY DEVELOPED TURBULENT FLOW IN CIRCULAR TUBES

The Dittus-Boelter equation

$$\mathrm{Nu_D} = 0.023\,\mathrm{Re_D}^{4/5}\,\mathrm{Pr}^n$$

where n = 0.4 for heating($T_s > T_b$) and 0.3 for cooling($T_s < T_b$). This equation is valid for

$0.7 \le \mathrm{Pr} \le 160$

$\mathrm{Re_D} \ge 10{,}000$

$L/D \ge 60$

with the fluid properties evaluated at the bulk mean temperature.

Correlation for Thermal Entry region

Eq. 35 apply only to fully developed turbulent flow in tubes. Nusselt after studying the experimental data in the range L/D = 10 to 400, recommended the following correlation for the entry region in turbulent flow

$$\mathrm{Nu_D} = 0.036\,\mathrm{Re_D}^{0.8}\,\mathrm{Pr}^{1/3}\left(\frac{D}{L}\right)^{0.055} \quad \text{for } 10 < \frac{L}{D} < 400$$

where the fluid properties are evaluated at the mean bulk temperature.

Example

Water at 50^{0}C enters a 1.5 cm dia and 3 m long tube with a velocity of 1 m/s. The tube wall is maintained at a constant temperature of 90^{0}C. Calculate the heat transfer coefficient and the total amount of heat transferred if the exit water temperature is 64^{0}C.

Solution

Given data

The mean bulk temperature = (50 + 64)/2 = 57^{0}C. The properties of water at the average bulk temperature are

ρ = 990 kg/m^3 , ν = 0.517 x 10^{-6} m^2/s

C_p = 4184 J/kg.K, k = 0.65 W/mK

Pr = 3.15

Type of flow

$$Re_D = \frac{uD}{\nu} = \frac{(1)(0.015)}{0.517x10^{-6}} = 29015 \text{ (> 2500 hence turbulent)}$$

Use of Dittus-Boelter equation

since L/D = 300/1.5 = 200 so the Dittus-Boelter Eq. can be used.

$$Nu_D = 0.023 Re_D^{4/5} Pr^{0.4} = 0.023(29015)^{4/5}(3.15)^{0.4} = 135.25$$

Heat transfer coefficient

Heat transfer coefficient

$$h = Nu_D.\frac{k}{D} = \frac{(135.25)(0.65)}{0.015} = 5860\, W/m^2K$$

Heat transferred

The rate of heat transferred

$$Q = h(\pi DL)\left(T_s - \frac{T_{b0} + T_{bi}}{2}\right)$$

$$= (5860)(\pi x0.015x3)(90 - 57) = 27.34\, kW$$

Example

Engine oil flows through a 50 mm diameter tube at an average temperature of 147^{0}C. The flow velocity is 80 cm/s. Calculate the average heat transfer coefficient if the tube wall is maintained at a temperature of 200^{0}C and it is 2 m long.

Solution

Given data

The properties of unused engine oil at 147^{0}C are

ρ = 812.1 kg/m^3, ν = 6.94 x 10^{-6} m^2/s, C_p = 2.427 kJ/kg.K, k = 0.133 W/mK, Pr = 103

Type of flow

$$Re_D = \frac{uD}{\nu} = \frac{(0.8)(0.05)}{6.94x10^{-6}} = 5763 > 2500 \text{ hence turbulent}$$

$$\frac{L}{D} = \frac{2000}{50} = 40$$

Since L/D = 40 and Re_D = 5763 the flow is not fully developed.

Nusselt number

Following eq may be used to calculate the Nusselt number

$$Nu_D = 0.036 Re_D^{0.6} Pr^{1/3} \left(\frac{D}{L}\right)^{0.055}$$

$$= 0.03695763)^{0.8} (103)^{1/3} \left(\frac{1}{40}\right)^{0.055} = 140.26$$

Heat transfer

$\therefore$ $$h = Nu_D \frac{k}{D} = \frac{(140.26)(0.133)}{0.05} = 373.1 \, W/m^2K$$

Example

Air at 2 bar and 40°C is heated as it flows through a 30 mm diameter tube at a velocity of 10 m/s. If the wall temperature is maintained at 100°C all along the length of tube, make calculations for the heat transfer per unit length of the tube. Proceed to calculate the increase in bulk temperature over one metre length of the tube.

Use the following correlation

$Nu = 0.023 \, Re^{0.8} \, Pr^{0.4}$

and take the following thermo-physical properties of air at the average film temperature of 70°C.

$p = 20.6 \times 10^{-6}$ *Ns/m*^{2} *;*

$c_p = 1.009$ *kJ/kg-deg*

$k = 0.0297$ *W/m-deg and*

$Pr = 0..694$

Solution

Density, Reynolds number and Nusselt number

$$\rho = \frac{p}{RT} = \frac{2x10^5}{287x(273+40)} = 2.226 \, kg/m^3$$

and $$Re = \frac{\rho vd}{\mu} = \frac{2.226x10x0.03}{20.6x10^{-6}} = 32417 > 2500, turbulent$$

Now $$Nu = \frac{hd}{k} = 0.023(32417)^{0.8}(0.694)^{0.4} = 80.697$$

Heat transfer coefficient

∴ $$h = Nu\frac{k}{d} = 80.697x\frac{0.0297}{0.03} = 79.89W/m^2K$$

Calculations for exit temperature

Let t_1 and t_2 denote the temperature of air at entrance and at exit of the heated section. This mean bulk temperature of the air is

$$t_b = \frac{t_1 + t_2}{2} = \frac{40 + t_2}{2}$$

Heat given by air

$$= m_a c_p \Delta t = \rho\frac{\pi}{4}d^2 V c_p \Delta t$$

$$= 2.226x\frac{\pi}{4}(0.03)^2 x10x1009x(t_2 - 40) = 15.868(t_2 - 40)$$

Convective heat flow from tube wall to air

$$= h(\pi dl)\Delta t$$

$$= 79.89(\pi x0.03x1)\left(100 - \frac{40 + t_2}{2}\right)$$

$$= 7.526\left(100 - \frac{40 + t_2}{2}\right)$$

Under steady state conditions, the heat gained by air equals the convective heat flow from tube wall to air . That is

$$15.868(t_2 - 40) = 7.526\left(100 - \frac{40 + t_2}{2}\right) = \frac{7.526}{2}(160 - t_2)$$

From this, $t_2 = 63^0$C

Now $Q = 15.868(63 - 40) = 364.96w/M$

Rise in bulk temperature = 63 - 40 = 23^0C

FREE CONVECTION VS. FORCED CONVECTION

Free convection	Forced convection
In free convection, the molecules move due to density and temperature variation.	In forced convection, the fluid molecules are forced to move by an external source.
The rate of heat transfer is lower.	The rate of heat transfer is higher.

No external equipment is required.	External equipment is necessary for convective heat transfer e.g. Pump, Blower, fan, etc.
The motion of molecules is comparatively slower.	Molecules of fluid are forced to move faster.
Equipment based on natural convection is larger in size.	The equipment based on forced convection is compact in size.
The flow of molecules cannot be controlled.	The flow of molecules can be controlled by controlling the fan, pump, or blower.
It has less overall heat transfer coefficient.	The overall heat transfer coefficient is higher.
E.g. movement of water molecules while boiling.	E.g. Movement of molecules due to the fan or blower.

ASSIGNMENT

Q.1. (AMIE W15, 17, 18, 19, 20, S19, 21, 10 marks): Differentiate between forced convection and free convection.

Q.2. (AMIE S20, 5 marks): Explain about heat transfer boundary layer.

Q.3. (AMIE S20, 5 marks): Explain about reduced viscosity and critical constants.

Q.4. (AMIE S16, 10 marks): Consider the flow of an incompressible Newtonian fluid between two coaxial cylinders. The surfaces of the inner and outer cylinders are maintained at $T = T_o$ and $T = T_b$, respectively. We can expect that T will be a function of r alone.

As the outer cylinder rotates, each cylindrical shell of fluid "rubs" against an adjacent shell of fluid. This friction between adjacent layers of the fluid produces heat; that is, the mechanical energy is degraded into thermal energy. The volume heat source resulting from this "viscous dissipation," which can be designated by S_v, appears automatically in the shell balance when we use the combined energy flux vector e.

If the slit width b is small with respect to the radius R of the outer cylinder, then the problem can be solved approximately by using the somewhat simplified system. That is, we ignore curvature effects and solve the problem in Cartesian coordinates. The velocity distribution is then $v_r = v_b(x/b)$, where $v_b = \Omega R$. Develop the temperature profile.

Q.5. (AMIE S17, W19, 5 marks): Derive Reynolds and Colburn analogies. If Pr = 1 then comment on the analogy.

Q.6. (AMIE W20, 3 marks): Depending upon the value of Prandtl's number which one grows faster - momentum layer or thermal boundary layer? Give justification.

Answer: If Pr > 1 the momentum or hydrodynamic boundary layer will increase more compared to the thermal boundary layer.

If Pr < 1 the thermal boundary layer will increase more compared to the momentum or hydrodynamic boundary layer.

If Pr = 1 The the thermal boundary layer and momentum or hydrodynamic boundary layer will increase at the same rate.

Q.7. (AMIE S17, W18, 19, 20, 15 marks): Through shell balance derive temperature distribution in a tube flow and hence prove that Nu = F(Re, Pr).

Q.8. (AMIE S18, 3 marks): Define the Grashof number. What is its physical significance?

Answer: Grashof number is a dimensionless number similar to Reynolds number. While Reynolds number is the ratio between inertial forces to viscous forces thus making Reynolds number useful for predicting the nature of the flow (Laminar, Turbulent or transition) thus making some approximations valid by knowing nature of the flow.

The same goes for Grashoff's number, however, it is defined as the ratio between the buoyancy forces and viscous forces in fluids. Why this is important? Because the buoyancy forces is what drives natural convection as the hot fluid goes up and the cold goes down, and the viscous force is what tries to stop it. By calculating the ratio you can predict if the natural convection is dominant or the forced conviction and then you can use simpler approaches to calculate the heat transfer.

Q.9. (AMIE S18, 19, S21, 7 marks): What is the energy equation for the laminar boundary layer on the flat plate? What assumptions are involved in the derivation of this equation?

Q.10. (AMIE S21, 5 marks): Water flows through a tube of radius 0.02 m and length 4 m at a velocity 1 m/s. It enters at a 20^0C, density and viscosity are 1000 kg/m^3 and 1 centipoise respectively. The wall of tube is maintained at 100^0C. If thermal conductivity of water is 0.502 J/s-m/K. Estimate the temperature of leaving water. Assume specific heat of water to be 4.2 J/gK and $f/2 = 0.046Re^{-0.2}$.

Answer: 373 K